MENIIT

NEET | IIT-JEE | FOUNDATION

Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAINS-2019

12-04-2019 Online (Morning)

IMPORTANT INSTRUCTIONS

1. The test is of 3 hours duration.
2. This Test Paper consists of **90 questions**. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of **Chemistry, Mathematics and Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
4. Out of the four options given for each question, only one option is the correct answer.
5. For each incorrect response 1 mark i.e. ¼ (one-fourth) marks of the total marks allotted to the question will be deducted from the total score. No deduction from the total score, however, will be made if no response is indicated for an item in the Answer Box.
6. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. One mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer box.
7. There is only one correct response for each question. Marked up more than one response in any question will be treated as wrong response and marked up for wrong response will be deducted accordingly as per instruction 6 above..

PART-A-CHEMISTRY

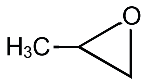
1. An ideal gas is allowed to expand from 1 L to 10 L against a constant external pressure of 1bar. The work done in kJ is :

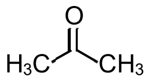
(1) –9.0 (2) +10.0 (3) –2.0 (4*) – 0.9

Sol. $W = P_{ext}(V_2 - V_1)$

2. The major product of the following addition reaction is :

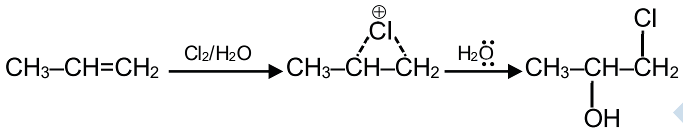
$H_3C - CH = CH_2 \xrightarrow{Cl_2/H_2O}$

(1) 

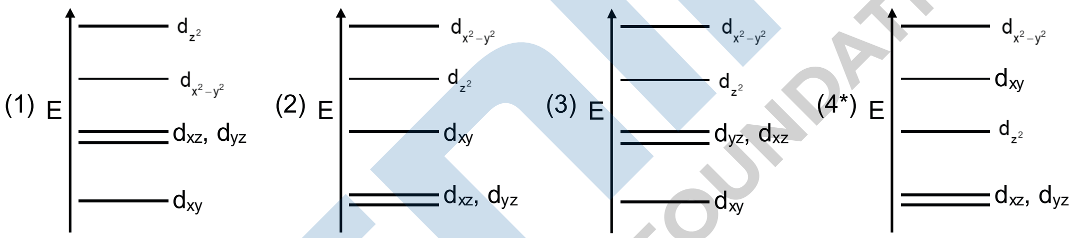
(2) 

(3*) $H_3C-CH(OH)-CH_2Cl$

(4) $CH_3-CH(Cl)-CH_2OH$

Sol. 

3. Complete removal of both the axial ligands (along the z-axis) from an octahedral complex leads to which of the following splitting patterns? (relative orbital energies not on scale).



Sol. Ligand filed exert mass repulsion along x, y axis as compared to Z-axis so $f_{x^2-y^2}$ and dxx will have increase in energy y

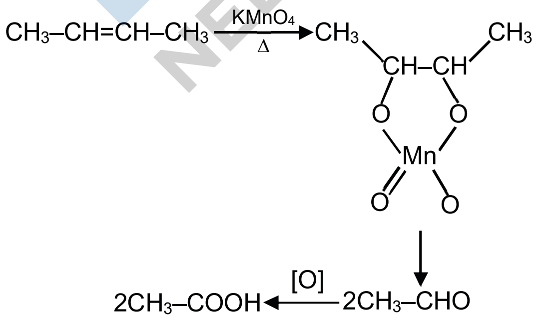
4. But-2-ene on reaction with alkaline $KMnO_4$ at elevated temperature followed by acidification will give:

(1) 2 molecules of CH_3CHO

(2) one molecule of CH_3CHO and one molecule of CH_3COOH

(3*) 2 molecules of CH_3COOH

(4) $CH_3-CH(OH)-CH(OH)-CH_3$

Sol. 

5. An organic compound 'A' is oxidized with Na_2O_2 followed by boiling with HNO_3.The resultant solution is then treated with ammonium molybdate to yield a yellow precipitate. Based on above observation, the element present in the given compound is :

(1) Sulphur (2*) Phosphorus (3) Nitrogen (4) Fluorine

Sol. Canary yellow ppt comes in test of po_4^{3-} ion.

6. The mole fraction of a solvent in aqueous solution of a solute is 0.8. The molality **(in mol kg^{-1})** of the aqueous solution is

(1*) 13.88 (2) 13.88×10^{-1} (3) 13.88×10^{-2} (4) 13.88×10^{-3}

Sol. $X_{solvent} = 0.8 = 8/10$

$N_{Total} = 10$, $n_{solutent} = 8$, $n_{solute} = 2$

Wt of solvent = 8×18

$$\text{Molality} = \frac{2 \times 1000}{8 \times 18}$$

7. The **correct** set of species responsible for the photochemical smog is :

(1) N_2, O_2, O_3 and hydrocarbons (2) N_2, NO_2 and hydrocarbons

(3) CO_2, NO_2, SO_2 and hydrocarbons (4*) NO, NO_2, O_3 and hydrocarbons

Sol. Smog photochemical is NO, NO_2, O_3.

8. An example of a disproportionation reaction is :

(1) $2NaBr + Cl_2 \rightarrow 2NaCl + Br_2$ (2) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

(3) $2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$ (4*) $2CuBr \rightarrow CuBr_2 + Cu$

Sol. $2Cu^{\oplus} \longrightarrow Cu^{+2} + Cu$

9. Given :

$Co^{3+} + e^- \rightarrow Co^{2+}$; $E^o = +1.81$ V

$Pb^{4+} + 2e^- \rightarrow Pb^{2+}$; $E^o = +1.67$ V

$Ce^{4+} + e^- \rightarrow Ce^{3+}$; $E^o = +1.61$ V

$Bi^{3+} + 3e^- \rightarrow Bi$; $E^o = +0.20$ V

Oxidizing power of the species will increase in the order :

(1) $Co^{3+} < Ce^{4+} < Bi^{3+} < Pb^{4+}$ (2) $Ce^{4+} < Pb^{4+} < Bi^{3+} < Co^{3+}$

(3) $Co^{3+} < Pb^{4+} < Ce^{4+} < Bi^{3+}$ (4*) $Bi^{3+} < Ce^{4+} < Pb^{4+} < Co^{3+}$

Sol. Lower the standard reduction potential, more the ability to get reduced higher the oxidizing power.

10. The correct sequence of thermal stability of the following carbonates is

(1) $MgCO_3 < SrCO_3 < CaCO_3 < BaCO_3$

(2) $BaCO_3 < SrCO_3 < CaCO_3 < MgCO_3$

(3*) $MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

(4) $BaCO_3 < CaCO_3 < SrCO_3 < MgCO_3$

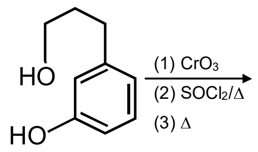
Sol. Smaller the size of cation, more polarisability high thermal decomposition of carbonates.

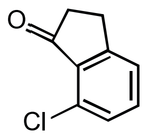
11. Enthalpy of sublimation of iodine is 24 cal g^{-1} at 200°C. If specific heat of I_2(s) and I_2(vap) are 0.055 and 0.031 cal $g^{-1}K^{-1}$ respectively, then enthalpy of sublimation of iodine at 250°C in cal g^{-1} is :

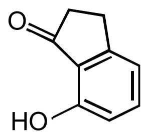
(1) 2.85 (2) 11.4 (3*) 22.8 (4) 5.7

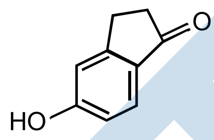
Sol. $\Delta C_P = \dfrac{\Delta H_2 - \Delta H_1}{T_2 - T_1}$

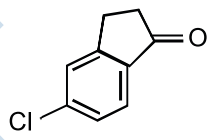
12. The major product of the following reaction

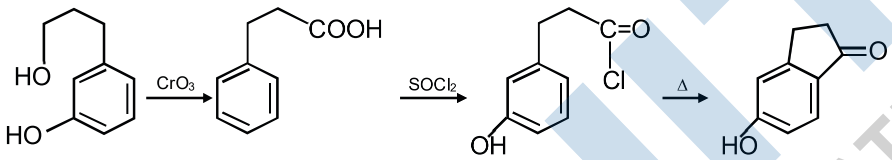


(1) 

(2) 

(3*) 

(4) 

Sol. 

13. Which of the following statements is not true about RNA ?

(1*) It has always double stranded α-helix structure

(2) It usually does not replicate

(3) It controls the synthesis of protein

(4) It is present in the nucleus of the cell

Sol. Not always double stranded α-helix.

14. The idea of froth floatation method came from a person X and this method is related to the process Y of ores. X and Y, respectively, are:

(1) fisher woman and concentration (2) washer man and reduction

(3) fisher man and reduction (4*) washer woman and concentration

Sol. Concentration (on dressing), i.e. washing of ore.

15. The group number, number of valence electrons, and valency of an element with atomic number 15, respectively, are

(1) 15, 6 and 2 (2) 16, 5 and 2 (3*) 15, 5 and 3 (4) 16, 6 and 3

Sol. 15 → $1s^2, 2s^2, 2p^6, 3s^2, 3p^3$} Period- III, Group-V

16. 5 moles of AB_2 weigh 125×10^{-3} kg and 10 moles of A_2B_2 weigh 300×10^{-3} kg. The molar mass of A(M_A) and molar mass of B(M_B) in kg mol^{-1} are :

(1) $M_A = 25 \times 10^{-3}$ and $M_B = 50 \times 10^{-3}$ (2*) $M_A = 5 \times 10^{-3}$ and $M_B = 10 \times 10^{-3}$

(3) $M_A = 50 \times 10^{-3}$ and $M_B = 25 \times 10^{-3}$ (4) $M_A = 10 \times 10^{-3}$ and $M_B = 5 \times 10^{-3}$

Sol. Mol. wt is of 1 mol

AB_2 $A + 2B = 25$

A_2B_2 $2A + 2B = 30$

17. The basic structural unit of feldspar, zeolites, mica, and asbestos is :

(1*) $(SiO_4)^{4-}$ (2) SiO_2 (3) $(SiO_3)^{2-}$ (4) $\left(\begin{matrix} R \\ | \\ Si-O \\ | \\ R \end{matrix}\right)_n$ (R = Me)

Sol. Basic unit is silicate $(SiO_4)^{4-}$

18. What is the molar solubility of $Al(OH)_3$ in 0.2 M NaOH solution ? Given that, solubility product of $Al(OH)_3$ $= 2.4 \times 10^{-24}$:

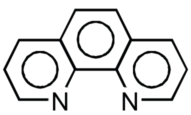
(1*) 3×10^{-22} (2) 12×10^{-23} (3) 3×10^{-19} (4) 12×10^{-21}

Sol. $Al(OH)_3 \rightleftharpoons Al^{3+} + 3OH^-$

S (3S + 0.2)

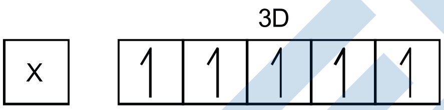
$2.4 \times 10^{-24} = s(3s + 0.2)^3$

19. The complex ion that will lose its crystal field stabilization energy upon oxidation of its metal to +3 state is -

(Phen =  and ignore pairing energy)

(1) $[Zn(phen)_3]^{2+}$ (2) $[Ni(phen)_3]^{2+}$ (3) $[Co(phen)_3]^{2+}$ (4*) $[Fe(phen)_3]^{2+}$

Sol. When Fe^{2+} oxidizes to Fe^{3+}



20. In the following reaction; $xA \longrightarrow yB$

$$\log_{10}\left[-\frac{d[A]}{dt}\right] = \log_{10}\left[\frac{d[B]}{dt}\right] + 0.3010$$

'A' and 'B' respectively can be :

(1*) C_2H_4 and C_4H_8 (2) n-Butane and Iso-butane

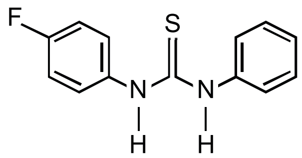
(3) C_2H_2 and C_6H_6 (4) N_2O_4 and NO_2

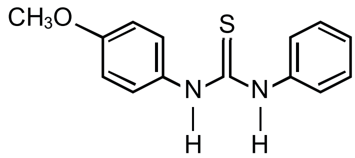
Sol. $$-\frac{1}{x}\left(\frac{dA}{dt}\right) = \frac{1}{y}\left(\frac{dB}{dt}\right)$$

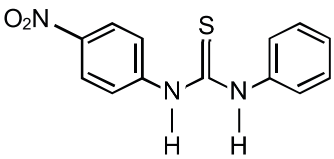
$$\log_{10}\left[-\frac{d(A)}{dt}\right] = \left[\log_{10}\left(\frac{dB}{dt}\right)\right] + \log\frac{x}{y}] \log\frac{x}{y} = \log 0.3$$

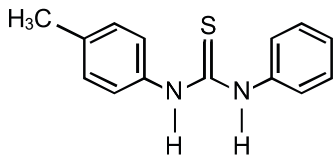
$$\left(\frac{x}{y} = 2\right)$$

21. The increasing order of the pK_b of the following compound is -

(A) 

(B) 

(C) 

(D) 

Options :

(1*) (B) < (D) < (A) < (C)

(2) (B) < (D) < (C) < (A)

(3) (C) < (A) < (D) < (B)

(4) (A) < (C) < (D) < (B)

Sol. CH_3O^-, CH_3^-, H is electron donating, But $-NO_2$ is electron withdrawing group.

22. An element has a face-centred cubic (fcc) structure with a cell edge of a. The distance between the centres of two nearest tetrahedral voids in the lattice is :

(1) a

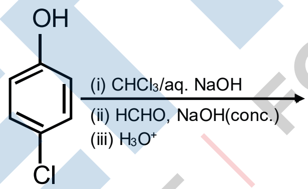
(2) $\frac{3}{2}a$

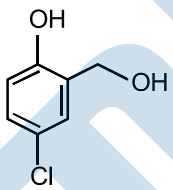
(3) $\sqrt{2}a$

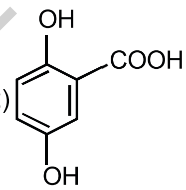
(4*) $\frac{a}{2}$

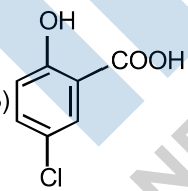
Sol. Two nearest tetrahedral voids are along one body-diagonal.

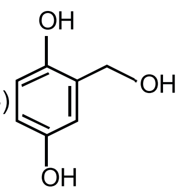
23. The major products of the following reaction are :



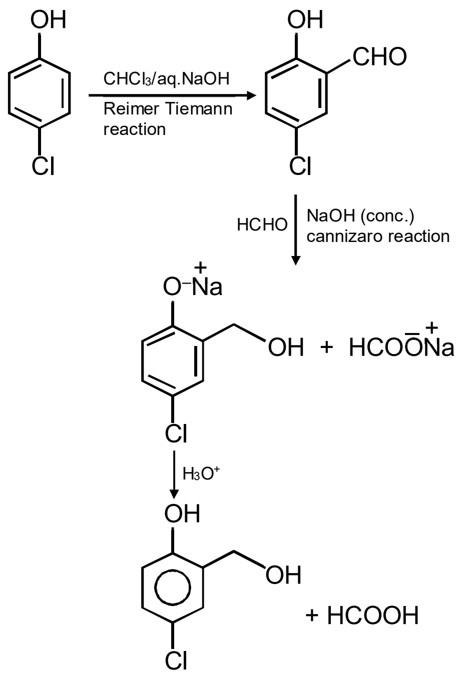
(1*)  and Formic acid

(2)  and Methanol

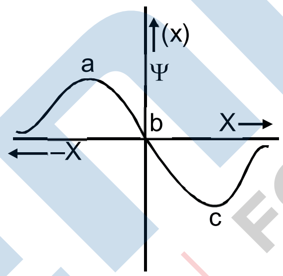
(3)  and Methanol

(4)  and Formic acid

Sol.

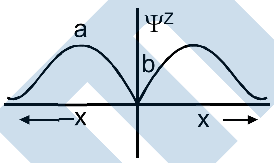


24. The electrons are more likely to be found :



(1) only in the region a
(2) only in the region c
(3) in the region a and b
(4*) in the region a and c

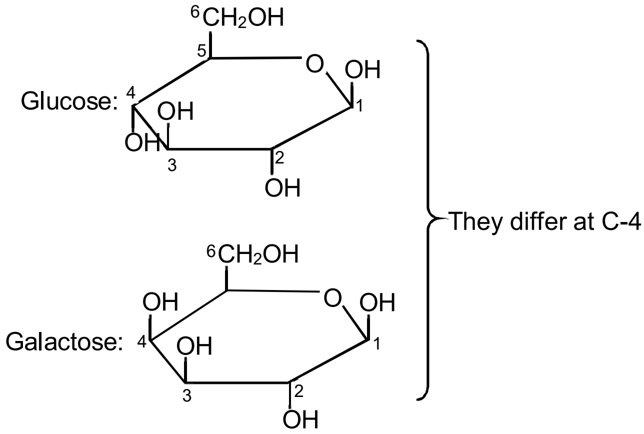
Sol. At a & c, probability of finding electron is maximum



25. Glucose and Galactose are having identical configuration in all the positions except position.

(1*) C–4
(2) C–5
(3) C–3
(4) C–2

Sol.



26. Peptization is a :

(1) process of converting a colloidal solution into precipitate

(2) process of converting soluble particles to form colloidal solution

(3*) process of converting precipitate into colloidal solution

(4) process of bringing colloidal molecule into solution

Sol. Peptization is process to form colloidal solution by adding peptizing agent.

27. The correct statement among the following is

(1*) $(SiH_3)_3N$ is planar and less basic than $(CH_3)_3N$

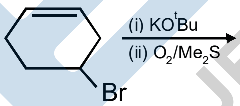
(2) $(SiH_3)_3N$ is planar and more basic than $(CH_3)_3N$

(3) $(SiH_3)_3N$ is pyramidal and more basic than $(CH_3)_3N$

(4) $(SiH_3)_3N$ is pyramidal and less basic than $(CH_3)_3N$

Sol. $p\pi$–$d\pi$ bonding in $N(SiH_3)_3$.

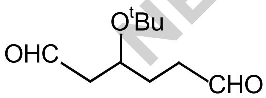
28. The major product(s) obtained in the following reaction is/are :



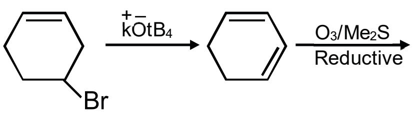
(1) 

(2) 

(3*) 

(4) 

Sol.



$CHO–CH_2–CH_2–CHO + OHC–CHO$

29. Which of the following is a thermosetting polymer?

(1*) Bakelite (2) Buna–N (3) PVC (4) Nylon 6

Sol. Thermosetting are cross-linked polymer.

30. The metal that gives hydrogen gas upon treatment with both acid as well as base is :

(1) iron (2*) zinc (3) mercury (4) magnesium

Sol. $Zn \xrightarrow{HCl} ZnCl_2 + H_2$

$\quad \searrow \xrightarrow{NaOH} Za_2ZnO_2 + H_2$

PART-B-MATHEMATICS

31. Let P be the point of intersection of the common tangents to the parabola y^2 = 12x and the hyperbola $8x^2 - y^2$ = 8. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio

(1) 13 : 11 (2) 2 : 1 (3) 14 : 13 (4*) 5 : 4

Sol. Tangents $y^2 = 12x \Rightarrow y = 2x + \frac{3}{m}$

$\frac{x^2}{1} - \frac{y^2}{8} = 1 \Rightarrow y = 2x + \frac{3}{m}$

Common tangent gives

$\therefore \frac{3}{m + \pm\sqrt{m^2 - 8}}$ $\frac{x^2}{1} - \frac{y^2}{8} = 1$

$m^4 - 8m^2 - 9 = 0$ e = 3

$m^4 - \pm 3$ ae = 3

$\therefore y = 3x + 1$ $P\left(-\frac{1}{3}, 0\right)$ $S = (3, 0)$

y = –3x – 1

P divides SS' om 5 : 4.

32. If B = $\begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of for which det (A) + 1 = 0, is

(1) 2 (2) 0 (3) –1 (4*) 1

Sol. $|B| = 5(-5) - 2\alpha(-\alpha) - 2\alpha$

$= 2\alpha2 - 2\alpha - 25$

1+ A = 0

$\alpha^2 - \alpha - 12 = 0$

Sum = 1

33. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to

(1) $\frac{21}{346}$ (2) $\frac{29}{358}$ (3*) $\frac{1}{12}$ (4) $\frac{7}{116}$

Sol. $375x^2 - 25x - 2 = 0$

$\alpha + \beta = \frac{25}{375}, \alpha\beta = \frac{-2}{375}$

$\Rightarrow$ ($\alpha + \alpha^2$ +……upto inf inite terms) + ($\beta + \beta^2$ +…….upto inf inite terms)

$= \frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\beta} = \frac{1}{12}$

34. If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane 2x + 3y – z + 13 = 0 at a point P and the plane 3x + y + 4z = 16 at a point Q, then PQ is equal to

(1) $2\sqrt{7}$ (2*) $2\sqrt{14}$ (3) $\sqrt{14}$ (4) 14

Sol. $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}=\lambda$

x = 3λ + 2, y = 2λ = 1, z = –λ + 1

Intersection with plane 2x + 3y – z + 13 = 0

2(3λ + 2) + 3(2λ –1) $(-\lambda+1)+13=0$

13λ + 13 = 0

λ = –1

∴ P(–1, – 3, 2)

Intersection with plane

3x + y + 4z = 16

3(3λ + 2) + (2λ – 1) +4(–λ + 1) = 16

λ = 1

Q(5, 1,0)

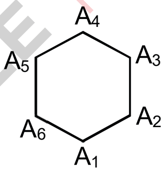
$PQ=\sqrt{6^2+4^2+2^2}=\sqrt{56}=2\sqrt{4}$

35. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is

(1*) $\frac{1}{10}$ (2) $\frac{3}{10}$ (3) $\frac{3}{20}$ (4) $\frac{1}{5}$

Sol. Only two equilateral triangle are possible $A_1A_2A_5$ and $A_2A_5A_6$

$\frac{2}{^6C_3}=\frac{2}{20}=\frac{1}{10}$



36. A 2m ladder leans against a vertical wall. It the top of the ladder begins to slide down the wall at the rate 25cm/sec, then the rate (in cm/sec) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1m above the ground is

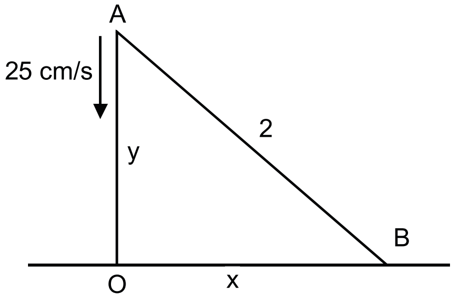
(1*) $\frac{25}{\sqrt{3}}$ (2) $\frac{25}{3}$

(3) 25 (4) $25\sqrt{3}$

Sol. $x^2+y^2=4\left(\frac{dy}{dt}=-25\right)$

$x\frac{dx}{dt}+y\frac{dy}{dt}=0$

$$\sqrt{3}\frac{dx}{dt} - 1(25) = 0$$



$$\frac{dx}{dt} = \frac{25}{\sqrt{3}} cm / sec$$

37. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is :

(1) $2\sqrt{2}$ (2) $\sqrt{2}$ (3*) 2 (4) 4

Sol. $x_1 + ... + x_1 = 44$

$x_5 + ... + x_{10} = 96$

$\bar{x} = 14, \sum x_i = 140$

$$\text{Variance} = \frac{\sum x_i^2}{n} - x^{-2} = 4$$

Standard deviation = 2

38. If the truth value of the statement $p \to (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively

(1) T, F, T (2) F, T, T (3*) T, T, F (4) T, F, F

Sol. $P \to (\sim q \vee r)$

$\sim p \vee (\sim q) r)$

$$\left.\begin{matrix} \sim p \to F \\ \sim q \to F \\ \sim r \to F \end{matrix}\right\} \Rightarrow \left.\begin{matrix} p \to T \\ q \to T \\ r \to F \end{matrix}\right\}$$

39. The equation $|z - i| = |z - 1|, \; i = \sqrt{-1}$, represents

(1) a circle of radius 1 (2) a circle of radius $\frac{1}{2}$

(3) the line through the origin with slope –1 (4*) the line through the origin with slope 1

Sol. $|z - i| = |z - 1|$

Gives y = x

40. Let a random variable X have a binomial distribution with mean 8 and variance 4. If $P(X \le 2) = \frac{k}{2^{16}}$, then k is equal to

(1*) 137 (2) 17 (3) 121 (4) 1

Sol. np = 8

npq = 4

$q = \frac{1}{2} \Rightarrow p = \frac{1}{2}$

n = 16

$p(x-1) = {}^{16}C_r\left(\frac{1}{2}\right)^{16}$

$p(x \le 2) = \frac{{}^{16}C_0 + {}^{16}C_1 + {}^{16}C_2}{2^{16}}$

$= \frac{137}{2^{16}}$

41. The equation y = sin x sin (x + 2) – $\sin^2$ (x + 1) represents a straight line lying in

(1) first, second and fourth quadrants (2) first, third and fourth quadrants

(3*) third and fourth quadrants only (4) second and third quadrants only

Sol. 2y = 2sin x sin(x + 2) – $2\sin^2$ (x + 1)

2y = cos2 – cos(2x + 2) – (1– cos(2x + 2))

= cos 2 – 1

$2y = -2\sin^2\frac{1}{2}$

$y = -\sin^2\frac{1}{2} \le 0$

42. For $x \in \left(0, \frac{3}{2}\right)$, let f (x) = $\sqrt{x}$, g(x) = tan x and h(x) = $\frac{1-x^2}{1+x^2}$. If ϕ (x) = ((hof)og) (x), then $\varphi\left(\frac{\pi}{3}\right)$ is equal to

(1) $\tan\frac{7\pi}{12}$ (2) $\tan\frac{\pi}{12}$ (3) $\tan\frac{5\pi}{12}$ (4*) $\tan\frac{11\pi}{12}$

Sol. $f(x) = \sqrt{x}, g(x) = \tan x, h(x) = \frac{1-x^2}{1+x^2}$

$fog(x) = \sqrt{\tan x}$

$hotog(x) = h\left(\sqrt{\tan x}\right) = \frac{1-\tan x}{1+\tan x}$

$= -\tan\left(\frac{\pi}{4} - x\right)$

$$\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$$

$$\phi\left(\frac{x}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$$

$$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$$

43. If the area (in sq. units) of the region {(x, y) : y^2 4x, x + y 1, x 0, y 0} is , then a – b is equal to

(1) $\frac{10}{3}$ (2) $-\frac{2}{3}$ (3*) 6 (4) $\frac{8}{3}$

Sol. $\{(x, y)\} : y^2 \geq 4x,\ x + y \leq 1,\ x \geq 0,\ y \geq 0\}$

$$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}dx + \frac{1}{2}\left(1-\left(3-2\sqrt{2}\right)\right)\left(1-\left(3-2\sqrt{2}\right)\right)$$

$$= \frac{2\left[x^{3/2}\right]_0^{3-2\sqrt{2}}}{3/2} + \frac{1}{2}\left(2\sqrt{2}-2\right)\left(2\sqrt{2}-2\right)$$

$$= \frac{8\sqrt{2}}{3} + \left(-\frac{10}{3}\right)$$

$$a = \frac{8}{3}, b = -\frac{10}{3}$$

a – b = 6

44. Let S_n denote the sum of the first n terms of an A.P.. If S_4 = 16 and S_6 = – 48, then S_{10} is equal to

(1) – 410 (2) – 380 (3*) – 320 (4) – 260

Sol. 2{2a + 3d} = 16

3(2a + 5d) = –48

2a + 3d = 8

2a + 5d = –16

d = –12

S_{10} = 5 {44 – 9 ×12}

= –320

45. The number of solutions of the equation $1+ \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is

(1*) 5 (2) 3 (3) 7 (4) 4

Sol. $1+ \sin^4 x = \cos^2 3x$

sin x = 0 and cos3x = 1

$0, 2\pi, -2\pi, -\pi, \pi$

46. For $x \in R$, let [x] denote the greatest integer x, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right] \text{ is}$$

(1) – 153 (2*) – 133 (3) – 131 (4) – 135

Sol. $$\underbrace{\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+.....+\left[\frac{1}{3}-\frac{66}{100}\right]}_{(-1)67}+\underbrace{\left[-\frac{1}{3}-\frac{67}{100}\right]+.....+\left[-\frac{1}{3}-\frac{99}{100}\right]}_{(-1)67}=-133$$

47. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through Q(4, 4) then PQ is equal to

(1) $\frac{\sqrt{221}}{2}$ (2*) $\frac{5\sqrt{5}}{2}$ (3) $\frac{\sqrt{61}}{2}$ (4) $\frac{\sqrt{157}}{2}$

Sol. $3x^2 + 4y^2 = 12$

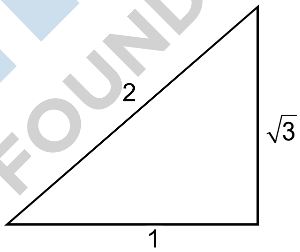
$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

$$x = 2\cos\theta,\ y=\sqrt{3}\sin\theta$$

Equation of normal is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$$2x\sin\theta-\sqrt{3}\cos\theta y=\sin\theta\cos\theta$$

Slope $\frac{2}{\sqrt{3}}\tan\theta=-2$ $\quad\therefore \tan\theta=-\sqrt{3}$



Equation of tangent is it passes through (4, 4)

$$3x\cos\theta+2\sqrt{3}\sin\theta y=6$$

$$12\cos\theta+8\sqrt{3}\sin\theta=6$$

$$\cos\theta=-\frac{1}{2},\ \sin\theta=\frac{\sqrt{3}}{2}\ \therefore\ \theta=120^\circ$$

$$P\left(-1,\frac{2}{2}\right),Q(4,4)$$

$$PQ=\frac{5\sqrt{5}}{2}$$

48. If $\int_0^{\frac{\pi}{2}}\frac{\cot x}{\cot x+\operatorname{cosec} x}dx=m(\pi+n)$, then $m \cdot n$ is equal to

(1) $\frac{1}{2}$ (2) 1 (3*) – 1 (4) $-\frac{1}{2}$

Sol. $$\int_0^{\pi/2}\frac{\cot x dx}{\cot x+\operatorname{cosec} x}$$

$$\int_0^{\pi/2}\frac{\cos x}{\cos x+1}=\int\frac{2\cos^2\frac{x}{2}-1}{2\cos^2\frac{x}{2}}$$

$$\int_0^{\pi/2}\left(1-\frac{1}{2}\sec^2\frac{x}{2}\right)dx$$

$$\left[x-\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

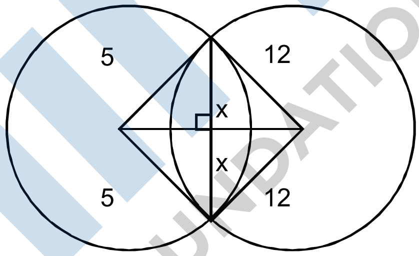
$\frac{1}{2}[\pi-2]$ $\qquad m=\frac{1}{2}, n=-2$

$mn = -1$

49. If the angle of intersection at a point where the two circles with radii 5cm and 12cm intersect is 90°, then the length (in cm) of their common chord is

(1*) $\frac{120}{13}$ (2) $\frac{60}{13}$ (3) $\frac{13}{2}$ (4) $\frac{13}{5}$

Sol. Let length of common chord = 2x



$\sqrt{25-x^2}+\sqrt{144-x^2}=13$

After solving $x=\frac{12\times5}{13}$

$2x=\frac{120}{13}$

50. If m is the minimum value of k for which the function $f(x)=x\sqrt{kx-x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when k = m, then the ordered pair (m, M) is equal to

(1) $\left(5, 3\sqrt{6}\right)$ (2) $\left(4, 3\sqrt{2}\right)$ (3*) $\left(4, 3\sqrt{3}\right)$ (4) $\left(3, 3\sqrt{3}\right)$

Sol. $f(x)=x\sqrt{kx-x^2}$

$f'(x)=\frac{3kx-4x^2}{2\sqrt{kx-x^2}}$

For increasing f (x) ≥ 0 $\qquad 3kx-4x^2\geq 0$

$kx-x^2\geq 0$ $\qquad 4x^2-3kx\leq 0$

$x^2-kx\leq 0$ $\qquad 4x\left(x-\frac{3k}{4}\right)\leq 0$

$x(x-k)\leq 0$ so $x\in[0, 3)$ $\qquad 3-\frac{3k}{4}\leq 0$

$+\text{ve}\boxed{x\geq 3}$ $\qquad k\geq 4$

minimum value of k is m = 4

$f(x)=x\sqrt{kx-x^2}$

$=3\sqrt{4\times3-3^2}=3\sqrt{3}, M=3\sqrt{3}$

51. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to :

(Here C is a constant of integration)

(1*) $\log_e\left|\frac{x^3+1}{x}\right|+C$ (2) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$

(3) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|}+C$ (4) $\log_e\frac{|x^3+1|}{x^2}+C$

Sol. $\int \frac{2x^3-1}{x^4+x}dx$

$\int \frac{2x-\frac{1}{x^2}}{x^2+\frac{1}{x}}dx$

$x^2+\frac{1}{x}=t$

$\left(2x-\frac{1}{x^2}\right)dx=dt$

$\int \frac{dt}{t}=\ell n(t)+C$

$=\ell n\left(x^2+\frac{1}{x}\right)+C$

52. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is

(1) $2^{20}+1$ (2) $2^{20}-1$ (3*) 2^{20} (4) 2^{21}

Sol. Since ${}^{21}C_0$ + ... + ${}^{21}C_{10}$ + ${}^{21}C_{11}$ + ... + ${}^{21}C_{21}$ = 2^{21}

$\Rightarrow$ but we have to select 10 objects and ${}^{21}C_0$ + ... + ${}^{21}C_{10}$ = ${}^{21}C_{11}$ + ... + ${}^{21}C_{21}$

(${}^{21}C_0$ +.......+${}^{21}C_{10}$) = 2^{20}

53. The value of $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$ is equal to

(1) $\pi-\cos^{-1}\left(\frac{33}{65}\right)$ (2) $\frac{\pi}{2}-\cos^{-1}\left(\frac{9}{65}\right)$ (3*) $\frac{\pi}{2}-\sin^{-1}\left(\frac{56}{65}\right)$ (4) $\pi-\sin^{-1}\left(\frac{63}{65}\right)$

Sol. $\sin^{-1}\left(\frac{12}{13}\right)-\sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$

$=\sin^{-1}\left(\frac{33}{65}\right)=\cos^{-1}\left(\frac{56}{65}\right)=\frac{\pi}{2}-\sin^{1}\left(\frac{56}{65}\right)$

54. Let $f : R \to R$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$.

If $\int_{6}^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim_{x\to 2} g(x)$ is equal to

(1) 12 (2*) 18 (3) 24 (4) 36

Sol. $\lim_{x\to 2} \frac{\int_{6}^{f(x)} 4t^3 dt}{x-2}$

$\lim_{x\to 2} \frac{4.f^3(x).f'(x)}{1}$

$-4f^3(2)f'(2) = 18$

55. Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is

(1) $4(2\hat{i} + 2\hat{j} + \hat{k})$ (2) $4(-2\hat{i} - 2\hat{j} + \hat{k})$ (3) $4(2\hat{i} + 2\hat{j} - \hat{k})$ (4*) $4(2\hat{i} - 2\hat{j} - \hat{k})$

Sol. $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$$= 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix}$$

$= 2(8\hat{i} - 8\hat{j} + 4\hat{k})$

$\text{Required vector} = \pm 12 \frac{(2\hat{i} - 2\hat{j} - \hat{k})}{3}$

$= \pm 4(2\hat{i} - 2\hat{j} - \hat{k})$

56. Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If value of y is 1 when x = 1, then the value of x for which y = 2, is

(1) $\frac{1}{2} + \frac{1}{\sqrt{e}}$ (2) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (3) $\frac{3}{2} - \sqrt{e}$ (4*) $\frac{3}{2} - \frac{1}{\sqrt{e}}$

Sol. $y^2 dx + x dy = \frac{dy}{y}$

$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

$IF = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$

$e^{\frac{1}{y}}.x = \int e^{\frac{1}{y}}.\frac{1}{y^3} dy + C$

$$xe^{\frac{1}{y}} = e^{\frac{1}{y}} + \frac{e - \frac{1}{y}}{y} + C$$

$$C = -\frac{1}{e}$$

$$x = \frac{3}{2} - \frac{1}{\sqrt{e}} \text{ when } y = 2$$

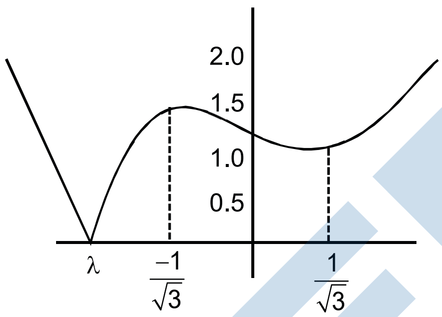
57. If the volume of parallelopiped formed by the vector $\hat{i} + \lambda\hat{j} + \hat{k}$, $\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} + \hat{k}$ and λ is minimum, then is equal to

(1) $-\frac{1}{\sqrt{3}}$ (2) $-\sqrt{3}$ (3) $\sqrt{3}$ (4*) $\frac{1}{\sqrt{3}}$

Sol. Volume of paralleopiped $= \begin{vmatrix} 1 & \lambda & 1 \\ 0 & 1 & \lambda \\ \lambda & 0 & 1 \end{vmatrix}$

$f(\lambda) = |\lambda^3 - \lambda + 1|$

Its graphs as follows



where $\lambda \approx -1.32$

For minimum value of volume of paralelopiped and corresponding value of λ ; the minimum value is zero, $\because$ cubic always has at least one real root.

Hence answer to the question must be root of cubic $\lambda^3 - \lambda + 1 = 0$. None of the options satisfies the cubic.

Hence Question must be Bonus.

58. If A is a symmetric matrix and B is a skew-symmetrix matrix such that A + B = $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to

(1) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$ (2) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$ (4*) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Sol. A = A', B = –B'

$$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \qquad \text{........(1)}$$

$$A'+B'=\begin{bmatrix}2 & 5\\ 3 & -1\end{bmatrix}$$

$$A+B=\begin{bmatrix}2 & 5\\ 3 & -1\end{bmatrix} \quad ....(2)$$

After adding equation (1) and (2)

$$A=\begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix}, \quad B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$$

59. The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is

(1) – 126 (2*) 84 (3) – 84 (4) 126

Sol. $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$

$(1 - x^2)(1 - x^3)^9$

${}^9C_6 = 84$

60. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at x = 0 is equal to

(1) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$ (2*) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (3) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$

Sol. $e^y = xy = e$ differentiate w.r.t. x

$$e^y\frac{dy}{dx}+x\frac{dy}{dx}+y=0$$

$$\frac{dy}{dx}\left(x+e^y\right)=-y, \left.\frac{dy}{dx}\right|_{(0,1)}=-\frac{1}{e} \text{ again differentiate w.r.t.x}$$

$$e^y.\frac{d^2y}{dx^2}+\frac{dy}{dx}.e^y.\frac{dy}{dx}+x.\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0$$

$$\left(x+e^y\right)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2.e^y+2\frac{dy}{dx}=0$$

$$e\frac{d^2y}{dx^2}+\frac{1}{e^1}e+2\left(-\frac{1}{e}\right)=0$$

$$\therefore \frac{d^2y}{dx^2}=\frac{1}{e^2}$$

PART-C-PHYSICS

61. An electromagnetic wave is represented by the electric field $\vec{E} = E_0\hat{n}\sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction of propagation , is :

(1) $\hat{s} = \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$ (2) $\hat{s} = \frac{-4\hat{j}+3\hat{k}}{5}$ (3) $\hat{s} = \frac{3\hat{j}-4\hat{k}}{5}$ (4*) $\hat{s} = \frac{4\hat{j}-3\hat{k}}{5}$

Sol. $\vec{E} = E_0\hat{n}\sin\left(\omega t + (6y - 8z) = E_0\hat{n}\sin\left(\omega t + \vec{k}\cdot\vec{r}\right)\right)$

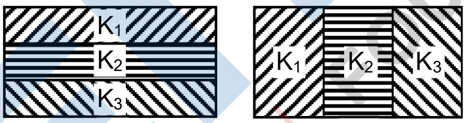
where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{k}\cdot\vec{r} = 6y - 8z$

$\Rightarrow \vec{k} = 6\hat{j} - 8\hat{k}$

Direction of propagation $\hat{s} = -\hat{k}$

$= \left(\frac{-3\hat{j}+4\hat{k}}{5}\right)$

62. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig. II. If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) :

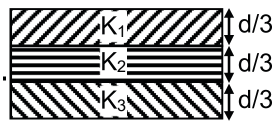


(I) (II)

(1) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$ (2) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

W (3) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$ (4*) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

Sol. $C_1 = \frac{3\varepsilon_0 AK_1}{d}$

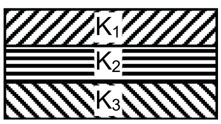
I.


$C_2 = \frac{3\varepsilon_0 AK_2}{d}$

$C_3 = \frac{3\varepsilon_0 AK_3}{d}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\Rightarrow E_{eq} = \frac{3\varepsilon_0 AK_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)}$ ....(i)

II.


$$c_1 = \frac{\varepsilon_0 K_1 A}{3d}$$

$$c_2 = \frac{\varepsilon_0 K_2 A}{3d}$$

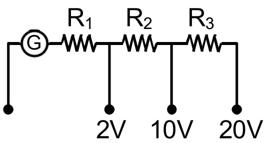
$$c_3 = \frac{\varepsilon_0 K_3 A}{3d}$$

$$C'_{eq} = C_1 + C_2 + C_3$$

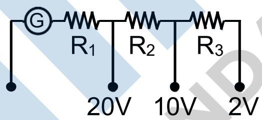
$$= \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \qquad ....(ii)$$

$$\text{Now, } \frac{E_1}{E_2} = \frac{\frac{1}{2}C_{eq} \cdot V^2}{\frac{1}{2}C'_{eq}V^2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_1K_2+K_2K_3+K_3K_1)}$$

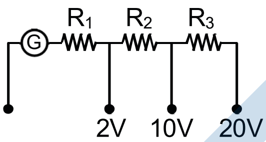
63. A galvanometer of resistance 100Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of 0–2 V, 0–10 V and 0–20 V. The appropriate circuit to do so is :

(1) 

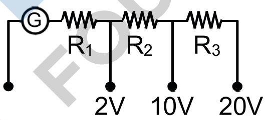
R_1 = 1900 Ω
R_2 = 8000 Ω
R_3 = 10000 Ω

(2) 

R_1 = 19900 Ω
R_2 = 9900 Ω
R_3 = 1900 Ω

(3) 

R_1 = 2000 Ω
R_2 = 8000 Ω
R_3 = 10000 Ω

(4*) 

R_1 = 1900 Ω
R_2 = 9900 Ω
R_3 = 19900 Ω

Sol. $20 \times 50 \times 10^{-6} = 10^{-3}$ Amp.

$$V_1 = \frac{2}{10^{-3}} = 100 + R_1$$

1900 = R_1

$$V_2 = \frac{10}{10^{-3}} = (2000 + R_2)$$

R_2 = 8000

$$V_3 = \frac{20}{10^{-3}} = 10 \times 10^3 R_3$$

64. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l ($l << L$), is close to :

(1*) $Mgl(1+\theta_0^2)$ (2) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$ (3) Mgl (4) $Mgl(1-\theta_0^2)$

Sol. Angular momentum conservation

$MV_0L = MV_1(L - \ell)$

$$V_1 = V_0\left(\frac{L}{L-\ell}\right)$$

$w_g + w_p = \Delta KE$

$$-mg\ell + w_p = \frac{1}{2}m\left(V_1^2 - V_0^2\right)$$

$$w_p = mg\ell + \frac{1}{2}mV_0^2\left(\left(\frac{L}{L-\ell}\right)^2 - 1\right)$$

$$= mg\ell + \frac{1}{2}mV_0^2\left(\left(1 - \frac{L}{L-\ell}\right)^2 - 1\right)$$

Now, $\ell << L$

By, Binomial approximation

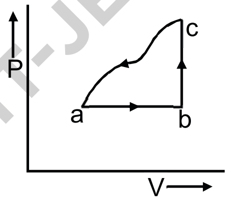
$$= mg\ell + \frac{1}{2}mV_0^2\left(\frac{2\ell}{L}\right)$$

$$W_P = mg\ell + mV_0^2\frac{\ell}{L}$$

Here, V_0 = maximum velocity = $\omega \times A = \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$

$$= mg\ell\left(1 + \theta_0^2\right)$$

65. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is –180J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is :



(1) 120 J (2) 140 J (3*) 130 J (4) 100 J

Sol.

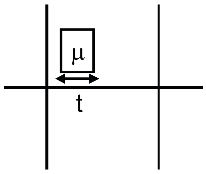
	ΔE	ΔW	ΔQ
ab			250
bc		0	60
ca	–180		

	ΔE	ΔW	ΔQ
ab	120	130	250
bc	60	0	60
ca	–180		

66. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) :

(1*) $\frac{\lambda}{(\mu-1)}$ (2) $\frac{\lambda}{2(\mu-1)}$ (3) $\frac{2\lambda}{(\mu-1)}$ (4) $\frac{\lambda}{(2\mu-1)}$

Sol.

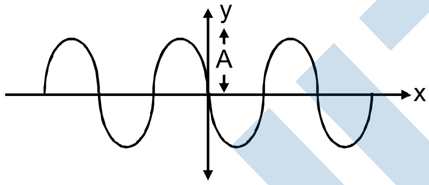


$\Delta x = (\mu - 1)t = 1\lambda$

For one maximum shift

$t = \frac{\lambda}{\mu - 1}$

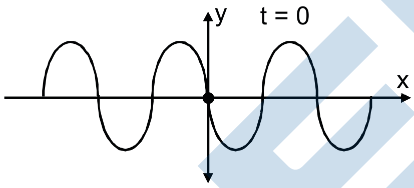
67. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin (kx - \omega t + \phi)$. Its snapshot at t = 0 is given in the figure:



For this wave, the phase ϕ is :

(1) $\frac{\pi}{2}$ (2) 0 (3) $-\frac{\pi}{2}$ (4*) π

Sol.



$y = A \sin (kx - wt + \phi)$

at x = 0, t = 0 and slope is negative

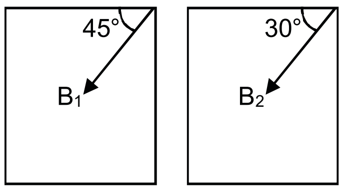
$\Rightarrow \phi = \pi$

68. A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by :

(1) 3.6 (2*) 0.7 (3) 1.8 (4) 2.2

Sol. $f_1 = \frac{1}{2\pi}\sqrt{\frac{\mu B_1 \cos 45^\circ}{I}}$ $f_2 = \frac{1}{2\pi}\sqrt{\frac{\mu B_2 \cos 30^\circ}{I}}$

$\frac{f_1}{f_2} = \frac{B_1 \cos 45^\circ}{B_2 \cos 30^\circ}$ $\therefore \frac{B_1}{B_2} \times 0.7$



69. When M_1 gram of ice at –10ºC (specific heat = 0.5 cal g^{-1}ºC^{-1}) is added to M_2 gram of water at 50ºC, finally no ice is left and the water is at 0ºC. The value of latent heat of ice, in cal g^{-1} is:

(1) $\frac{5M_2}{M_1} - 5$ (2*) $\frac{50M_2}{M_1} - 5$ (3) $\frac{50M_2}{M_1}$ (4) $\frac{5M_1}{M_2} - 50$

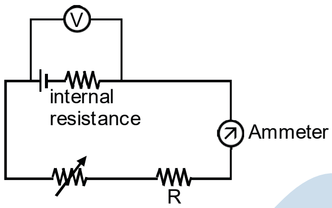
Sol. Heat lost = Heat gain

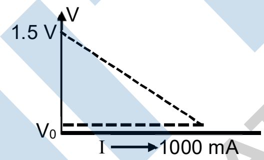
$\Rightarrow M_2 \times 1 \times 50 = M_1 \times 0.5 \times 10 + M_1,L_f$

$\Rightarrow \quad L_f = \frac{50M_2 - 5M_1}{M_1}$

$= \frac{50M_2}{M_1} = 5$

70. To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained:





If V_0 is almost zero, identify the correct statement:

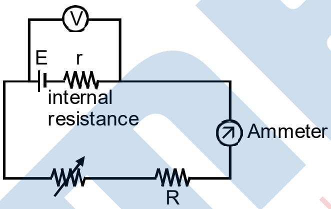
(1) The value of the resistance R is 1.5 Ω

(2*) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

(3) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA.

(4) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

Sol.



V = E – Ir

When V = V_0 = 0 · 0 = E – Ir

∴ E = r

When I = 0, V = E = 1.5 V

∴ r = 1.5 Ω

71. Which of the following combinations has the dimension of electrical resistance (ϵ_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum) ?

(1) $\frac{\varepsilon_0}{\mu_0}$ (2*) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (3) $\frac{\mu_0}{\varepsilon_0}$ (4) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$

Sol. $[\varepsilon_0] = M^{-1} L^{-3} T^4 A^2$

$[\mu_0] = M L T^{-2} A^{-2}$

$[R] = M L^2 T^{-3} A^{-2}$

$$[R] = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$$

72. At 40ºC, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40ºC to 20ºC it regains its original length of 0.2 m. The value of M is close to : (Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/ºC and 10^{11} N/m^2, respectively; g = 10 ms^{-2})

(1) 1.5 kg (2) 0.9 kg (3) 0.5 kg (4*) 9 kg

Sol. $Mg = \left(\frac{Ay}{\ell}\right)\Delta\ell$

$Mg = (Ay)\alpha\Delta T = 2\pi$

It is closest to 9.

73. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength λ, energy E = $\frac{1240\,eV}{\lambda\,(\text{in nm})}$:

(1) n = 6 (2) n = 7 (3) n = 4 (4*) n = 5

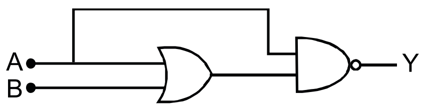
Sol. $\frac{1}{\lambda} = R\left(\frac{1}{M^2} - \frac{1}{n^2}\right)z^2$

$$\frac{1}{1085} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)2^2$$

$\therefore$ m = 2

$\therefore$ n = 5

74. The truth table for the circuit given in the fig. is:



(1)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(3*)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

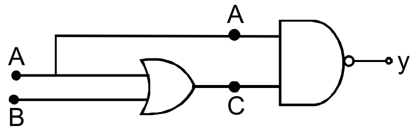
(4)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

Sol. C = A + B

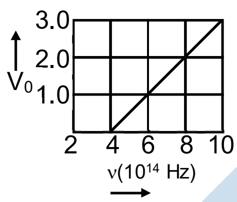
and $y = \overline{A.C}$

A	B	$C = (A + B)$	A.C.	$y = \overline{A.C.}$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0



75. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be :

(Given : Planck's constant (h) = 6.63 × 10^{-34} Js, electron charge e = 1.6 × 10^{-19} C)



(1) 1.95 eV (2) 2.12 eV (3) 1.82 eV (4*) 1.66 eV

Sol. $h\nu = \phi + ev_0$

$$v_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

v_0 is zero for $\nu = 4 \times 10^{14}$ Hz

$$0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

$\Rightarrow \phi = h\nu$

$$= \frac{6.63 \times 10^{-33} \times 4 \times 10^{14}}{1.6 \times 10^{-19}} = 1.66 \text{ eV}$$

76. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume ? (R = 8.3 J/mol K)

(1) 19.7 J/mol K (2) 21.6 J/mol K (3*) 17.4 J/mol K (4) 15.7 J/mol K

Sol. $$f_{mix} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2} = \frac{2 \times 3 + 3 \times 5}{5} = \frac{21}{5}$$

$$C_v = \frac{fR}{5} = \frac{21}{5} \times \frac{R}{2} = 17.4 \text{ J/mol K}$$

77. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10 \text{ ms}^{-2}$) :

(1) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} \text{ms}^{-1}$

(2*) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3} \text{ms}^{-1}$

(3) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} \text{ms}^{-1}$

(4) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5} \text{ms}^{-1}$

Sol. Equation of trajectory is given as

$y = 2x - 9x^2$ ...(A)

Comparing with equation:

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}\cdot x^2 \quad .....(B)$$

We get, $\tan\theta = 2$

$$\therefore \cos\theta = \frac{1}{\sqrt{5}}$$

$$\text{Also, } \frac{g}{2u^2\cos^2\theta} = g$$

$$\Rightarrow \frac{10}{2\times 9\times\left(\frac{1}{\sqrt{5}}\right)^2} = u^2 \quad ; \quad u^2 = \frac{25}{9}$$

$$\Rightarrow u = \frac{5}{3}\,m/s$$

78. A point dipole $\vec{P} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively: (Take V = 0 at infinity) :

(1*) $0, \frac{-\vec{p}}{4\pi\epsilon_0 d^3}$

(2) $\frac{|\vec{p}|}{4\pi\epsilon_0 d^2}, \frac{-\vec{p}}{4\pi\epsilon_0 d^3}$

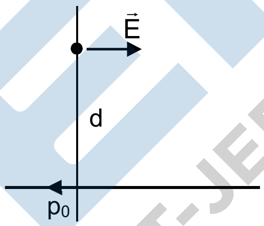
(3) $0, \frac{\vec{p}}{4\pi\epsilon_0 d^3}$

(4) $\frac{|\vec{p}|}{4\pi\epsilon_0 d^2}, \frac{\vec{p}}{4\pi\epsilon_0 d^3}$

Sol. V = 0

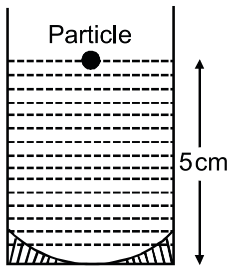
$$E = -\frac{K\vec{P}}{r^3}$$

$$= -\frac{\vec{p}}{4\pi\varepsilon_0 d^3}$$



79. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to :

(Refractive index of water = 1.33)



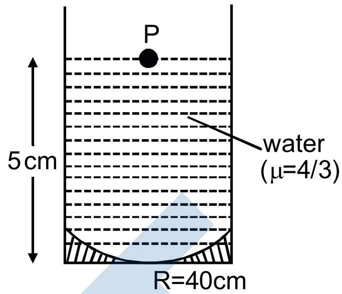
(1) 6.7 cm (2*) 8.8 cm (3) 11.7 cm (4) 13.4 cm

Sol. Light incident from particle P will be reflected at mirror.

$$u = -5\text{cm},\ f = m - \frac{R}{2} = -20\text{cm}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad ; \quad \boxed{v_1 = +\frac{20}{3}}\text{cm}$$

This image will act as object for light getting refracted at water surface.



$$\text{So, object distance } d = 5 + \frac{20}{3} = \frac{35}{3}\text{cm}$$

Below water surface.

After refraction, final image is at

$$d' = d\left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{35}{3}\right)\left(\frac{1}{4/3}\right)$$

$$= \frac{35}{4} = 8.75 \text{ cm}$$

≈ 8.8 cm

80. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is:

(1*) R/2g (2) R/4g (3) R/g (4) 2R/g

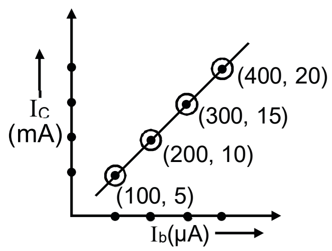
Sol. Range will be same for time t_1 and t_2, so angles of projection will be 'θ' & '90° – θ'

$$t_1 = \frac{2u\sin\theta}{g} \quad t_2 = \frac{2u\sin(90^\circ - \theta)}{g} \text{ and } R = \frac{\mu^2 \sin 2\theta}{g}$$

$$t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g} = \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]$$

$$= \frac{2R}{g}$$

81. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 k Ω respectively, is shown in the figure. The Voltage and Power gain, are respectively :



(1*) 5×10^4, 2.5×10^6 (2) 5×10^4, 5×10^5 (3) 5×10^4, 5×10^6 (4) 2.5×10^4, 2.5×10^6

Sol. $V_{gain} = \left(\frac{\Delta \ell_C}{\Delta \ell_B}\right)\frac{R_{out}}{R_{in}} = 5 \times 10^4$

$= \frac{1}{20} \times 10^8 = 5 \times 10^4$

$P_{gain} = \left(\frac{\Delta \ell_c}{\Delta \ell_b}\right)(V_{gain})$

$= \left(\frac{5 \times 10^{-3}}{100 \times 10^{-6}}\right)(5 \times 10^4)$

$= 2.5 \times 10^6$

82. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2) :

(1) 2×10^{-6} C (2*) 3×10^{-5} C (3) 7×10^{-6} C (4) 4×10^{-5} C

Sol. $B = \frac{\mu_0 i}{2R} = \frac{\mu_0 q_\omega}{2R\ 2\pi}$

$\Rightarrow q = 3 \times 10^{-5}$ C

83. The value of numerical aperature of the objective lens of a microscope is 1.25. If light of wavelength 5000 Å is used, the minimum separation between two points, to be seen as distinct, will be :

(1*) 0.24 μm (2) 0.38 μm (3) 0.12 μm (4) 0.48 μm

Sol. Numerical aperature of the microscope is given as

$NA = \frac{0.61\lambda}{d}$

Where d = minimum sparaton between two points to be seen as distinct

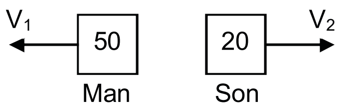
$d = \frac{0.61\lambda}{NA} = \frac{(0.61) \times (5000 \times 10\ m^{-10})}{1.25} = 2.4 \times 10^{-7} m$

$= 0.24\ \mu m$

84. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is :

(1) 0.28 ms^{-1} (2) 0.47 ms^{-1} (3) 0.14 ms^{-1} (4*) 0.20 ms^{-1}

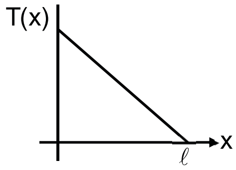
Sol.

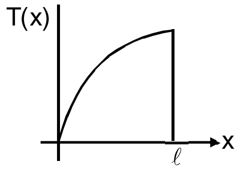


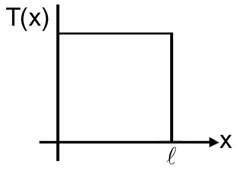
$\Rightarrow 0 = 50V_1 - 20V_2$ and $V_1 + V_2 = 0.7$

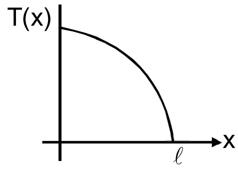
$\Rightarrow V_1 = 0.2$

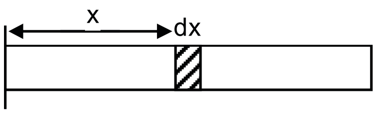
85. A uniform rod of length ℓ is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T(x) at a distance x from the axis, then which of the following graphs depicts it most closely?

(1) 

(2) 

(3) 

(4*) 

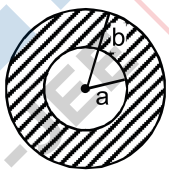
Sol. 

$$T = \int_{x=x}^{x=\ell} dm\omega^2 x = \int_{x=x}^{x=\ell} \frac{m}{\ell} dx\omega^2 x \; T$$

$$= \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$$

$$T = \frac{m\omega^2}{2\ell}\left(\ell^2 - x^2\right)$$

86. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is :

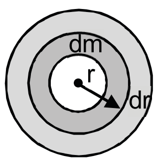


(1) $\frac{a+b}{3}$ (2*) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (3) $\sqrt{\frac{a^2+b^2+ab}{3}}$ (4) $\frac{a+b}{2}$

Sol. $dI = (dm)r^2$

$= (\sigma\, dA)r^2$

$$= \left(\frac{\sigma_0}{r} 2\pi dr\right) r^2 = (\sigma_0 2\pi) 0r^2 dr$$



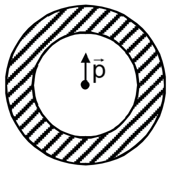
$$I = \int DI = \int_a^b \sigma_0 2\pi r^2 dr$$

$$= \sigma_0 2\pi\left(\frac{b^3 - a^3}{3}\right)$$

$M = \int dm = \int \sigma dA$

$= \sigma_0 2\pi \int_a^b dr$

87. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge At its centre is a dipole $\vec{p}$ as shown. In this case :



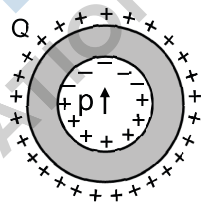
(1) Surface charge density on the inner surface of the shell is zero everywhere

(2) Surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$

(3*) Electric field outside the shell is the same as that of a point charge at the centre of the shell.

(4) Surface charge density on the outer surface depends on $|\vec{p}|$

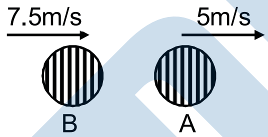
Sol. Total charge of dipole = 0, so charge induced onoutside surface = 0.
But due to non uniform electric field of dipole, the charge induced on
Inner surface is non zero and non uniform.
So, for any observer outside the shell, the resultant electric field is due
to Q uniformly distributed on outer surface only and it is equal to.



88. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to : (Speed of sound in water = 1500 ms^{-1})

(1*) 502 Hz (2) 507 Hz (3) 504 Hz (4) 499 Hz

Sol.



f_0 = 500 Hz frequency received by B again =

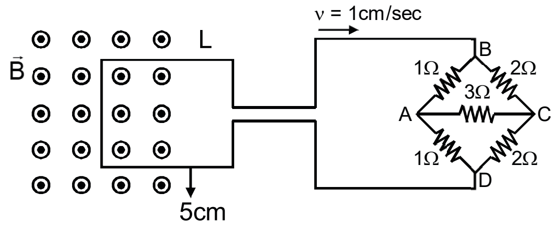
$\longleftarrow$ 1500

(B) (A) & $\Rightarrow$

7.5 m/s $\longrightarrow$ $\longrightarrow$ 5 m/sec

$$f_2 = \left(\frac{1500+7.5}{1500+5}\right) \times \left(\frac{1500-5}{1500-7.5}\right) f_0 = 502 \text{ Hz}$$

89. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cms^{-1}. At some instant, a part of L is in a uniform magnetic field of 1T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to :



(1) 115 μA (2*) 170 μA (3) 150 μA (4) 60 μA

Sol. Since it is a balanced wheatstone bridge, its equivalent resistance $= \frac{4}{3}\Omega$

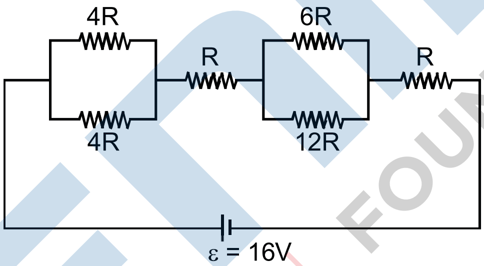
$\varepsilon = B\ell v = 5 \times 10^{-4}$ V

So total resistance

$$E = \frac{4}{3} + 1.7 \approx 3\Omega$$

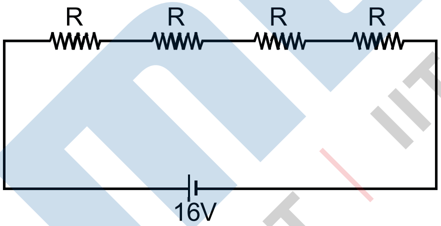
$$\therefore \quad i = \frac{\varepsilon}{R} \approx 166\mu A \approx 170\mu A$$

90. The resistive network shown below is connected to a D.C. source of 16V. The power consumed by the network is 4 Watt. The value of R is :



(1*) 8 Ω (2) 1 Ω (3) 6 Ω (4) 16 Ω

Sol.



$$P = \frac{16^2}{8R} = 4$$

$\therefore$ R = 8Ω